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FREQUENCY SPECTRA AND MODES OF FREE VIBRATIONS OF DOUBLY PERIODIC SYSTEMS

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An analog of the finite element method is proposed for the solution of natural vibrations problems for doubly-periodic systems. The approximate solution is constructed for each separate element. The infuence of adjacent elements is taken into account by the introduction of force factors and matching conditions. Numerical examples are analyzed.

1. Let the doubly-periodic system be generally referred to an oblique $0x_1x_2$ coordinate system so that the properties of the system are repeated for a displacement a_1 along the $0x_1$ axis and a_2 along $0x_2$. Let us consider the vibrations of a single element bounded by the lines $x_1' = 0$, $x_1' = a_1$, $x_2' = 0$, $x_2' = a_2$ in a local coordinate system. Let us represent the dicplacement vector for the vibrations mode as a series expansion in a system of coordinate functions

$$\mathbf{u}^{(kl)} = \sum_{n=1}^{N} C_n^{(kl)} \mathbf{v}^{(n)} \tag{1.1}$$

Here kl is the number of the element under consideration in the double numbering.

Let us consider the equations of motion and the conditions of element matching result in an equation in the coefficient vector $\mathbf{C}$

$$PC^{(kl)} + QC^{(k+1,\,l)} + RC^{(k,\,l+1)} = 0 \tag{1.2}$$

The matrix components P, Q, R are independent of the number of the element because of the periodicity of the system properties.

It can be shown that the general solution of (1.2) is representable, analogously to the case of periodic structures [1, 2], as the sum of solutions of the type

$$\mathbf{C}^{(k,\,l)} = \lambda_1{}^k\lambda_2{}^l\mathbf{C} \tag{1.3}$$

Let P, Q, R be nonsingular matrices. For some fixed value of the complex λ_2 let us consider the equation

$$(P + \lambda_1 Q + \lambda_2 R)\,\mathbf{v} = 0 \tag{1.4}$$

We hence obtain N eigenvalues $\lambda_{1n}(\lambda_2)$ and N eigenvectors $\mathbf{v}_n(\lambda_2)$.

Let l run through the values from 1 to L. Let us arbitrarily select L distinct values of $\lambda_2 : \lambda_{21}, \ldots, \lambda_{2L}$. Let us represent $\mathbf{C}^{(kl)}$ as the sum

$$\mathbf{C}^{(kl)} = \sum_{n=1}^{N}\sum_{m=1}^{L} \lambda_{2m}^{l} f_{nm}^{(k)} \mathbf{v}_n(\lambda_{2m}) \tag{1.5}$$

The numbers f_{nm} are hence found uniquely. Substituting (1.5) into (1.2), and taking account of (1.4), we obtain

$$\mathbf{C}^{(k+1,\,l)} = \sum_{n=1}^{N}\sum_{m=1}^{L} \lambda_{1n}(\lambda_{2m})\,\lambda_{2m}^{l} f_{nm}^{(k)} \mathbf{v}_n(\lambda_{2m}) \tag{1.6}$$

There hence results that in the general case numbers f_{nm} can be found for arbitrary k and l such that the expansion

$$\mathbf{C}^{(k,\,l)} = \sum_{n=1}^{N}\sum_{m=1}^{L} \lambda_{1nm}^{k}\lambda_{2m}^{l} f_{nm}\mathbf{v}(\lambda_{2m})$$

representable as a sum of solutions of the type (1.3), will be valid.

Let us prove the uniqueness of determining $f_{nm}^{(k)}$. To do this, let us represent (1.5) as

$$\mathbf{C}^{(kl)} = \sum_{m=1}^{L} \lambda_{2m}^{l} \sum_{n=1}^{N} f_{nm}^{(k)}\mathbf{v}_{nm}$$

Let us multiply the left and right sides by the vector φ. Because the λ_{2m} are distinct, we obtain that the determinant $\det \|\lambda_{2m}{}^l\| \neq 0$. Therefore,

$$\sum_{n=1}^{N} f_{nm}^{(k)}\mathbf{v}_{nm}\cdot\varphi = \sum_{l=1}^{L}\xi_{lm}\mathbf{C}^{(kl)}\cdot\varphi \tag{1.7}$$

$$\xi_{lm} = \xi_{lm}(\lambda_{21}, \lambda_{22}, \ldots, \lambda_{2L})$$

can be found uniquely.

Up to now, the vector φ has been arbitrary. Let us now consider the vector φ to be defined so that the conditions $\mathbf{v}_{nm}\cdot\varphi_p = \delta_{pn}$ are satisfied for fixed m. Because of the completeness and minimality of the system of vectors $\mathbf{v}_{nm}$ for fixed m, these equations uniquely define the vector φ. Under these conditions (1.7) yields

$$f_{nm}^{(k)} = \sum_{l=1}^{L} \xi_{lm} C^{(kl)} \cdot \varphi_{nm}$$

which indeed proves the uniqueness required.

The values of λ_1, λ_2, equal to one in absolute value, correspond to the solution of the natural vibrations problem for an unbounded doubly-periodic system. In the case of a bounded system, conditions for the boundary elements must be used to construct the frequency equation.

2. Let us apply the Rayleigh-Ritz method to find the solution of the system (1.2). Let us seek the components of the displacement $u_j^{(kl)}$ as the expansion

$$u_j^{(kl)} = \sum_{m=1}^{M} C_{jm}^{(kl)} f_{jm}(x_1', x_2') \qquad (2.1)$$

Let us represent the generalized forces or stresses acting on the edges $x_2' = 0$, $x_2' = a_2$ as the truncated series

$$\sigma_{2,j}^{(kl)} = \sum_{n=1}^{N} B_{2jn}^{(kl)} \varphi_{jn}(x_1'), \qquad \sigma_{2,j}^{(k,l+1)} = \sum_{n=1}^{N} B_{2jn}^{(k,l+1)} \varphi_{jn}(x_1') \qquad (2.2)$$

Let us analogously expand the generalized forces or stresses acting on the edges $x_1' = 0$, $x_1' = a_1$

$$\sigma_{1,j}^{(kl)} = \sum_{p=1}^{P} B_{1jp}^{(kl)} \psi_{jp}(x_2'), \qquad \sigma_{1,j}^{(k+1,l)} = \sum_{p=1}^{P} B_{1jp}^{(k+1,l)} \psi_{jp}(x_2') \qquad (2.3)$$

According to Rayleigh-Ritz method, it must be required that the following functional

$$J = (\mathbf{u} \cdot \mathbf{u})_1 - \omega^2 (\mathbf{u} \cdot \mathbf{u})_2 + (\boldsymbol{\sigma} \cdot \mathbf{u})_3 \qquad (2.4)$$

takes on a stationary value for the element under consideration.

The scalar product $(\mathbf{u} \cdot \mathbf{u})_1$ is generated by the strain potential energy, the expression $(\mathbf{u} \cdot \mathbf{u})_2$ corresponds to the kinetic energy, and the product $(\boldsymbol{\sigma} \cdot \mathbf{u})_3$ expresses the potential energy of the external forces.

Substituting (2.1) – (2.3) into (2.4) and equating the derivative $\partial J / \partial C_{iq}^{(kl)}$, we obtain

$$\begin{aligned} &\sum_j \sum_m C_{jm}^{(kl)} [(f_{jm}, f_{iq})_1 - \omega^2 (f_{jm}, f_{iq})_2] + \\ &\sum_j \sum_n [B_{2jn}^{(kl)} (\varphi_{jn}, f_{ig})_{320} + B_{2jn}^{(k,l+1)} (\varphi_{jn}, f_{ig})_{321}] + \\ &\sum_j \sum_p [B_{1jp}^{(kl)} (\psi_{jp}, f_{iq})_{310} + B_{1jp}^{(k+1,l)} (\psi_{jp}, f_{iq})_{311}] = 0 \end{aligned} \qquad (2.5)$$

Here the first digit in the triplet subscript of the scalar products denotes the number of the scalar product, while the other two denote the number of the edge sections.

We obtain additional relationships from the condition of equal displacements along the boundaries of the elements

$$\begin{aligned} &\sum_j \sum_m [C_{jm}^{(k,l+1)} (\varphi_{in}, f_{jm})_{320} - C_{jm}^{(kl)} (\varphi_{in}, f_{jm})_{321}] = 0 \\ &\sum_j \sum_m [C_{jm}^{(k+1,l)} (\psi_{ip}, f_{jm})_{310} - C_{jm}^{(kl)} (\psi_{ip}, f_{jm})_{311}] = 0 \end{aligned} \qquad (2.6)$$

Formulas (2.5), (2.6) yield a system of $J(M+N+P)$ linear algebraic equations. The solution of this system can be found as the sum of solutions of the form

$$C_{jmn}^{(kl)} = \lambda_1{}^k\lambda_2{}^l C_{jmn}, \qquad B_{\alpha jq}^{(kl)} = \lambda_1{}^k\lambda_2{}^l B_{\alpha iq} \tag{2.7}$$

3. As an illustrative example, let us examine the vibrations of an infinite plate supported on a rectangular lattice of diaphragms which are absolutely stiff in their planes and absolutely flexible out of the planes (hinge lines). Setting $P = N$, $M = N^2$, let us select the following functions as f_m, φ_n, ψ_p :

$$f_m = \varphi_n\psi_p, \quad \varphi_n = \sin\frac{\pi n x_1}{a_1}, \quad \psi_p = \sin\frac{\pi p x_2}{a_2}$$

The system (2.5) becomes

$$\frac{1}{4}\rho h a_1 a_2(\omega_{np}^2 - \omega^2)C_{np}^{(kl)} + \frac{\pi a_1 p}{2a_2}[B_{2n}^{(kl)} - (-1)^p B_{2n}^{(k,l+1)}] + \tag{3.1}$$

$$\frac{\pi a_2 n}{2a_1}[B_{1p}^{(kl)} - (-1)^p B_{1n}^{(k+1,l)}] \doteq 0$$

$$\omega_{np}^2 = \pi^4 D/\rho h\,[(n/a_1)^2 + (p/a_2)^2]^2$$

The matching conditions yield

$$\sum_{n=1}^{N} n\,[(-1)^n C_{np}^{(kl)} - C_{np}^{(k+1,l)}] = 0, \qquad \sum_{n=1}^{N} p\,[(-1)^p C_{np}^{(kl)} - C_{np}^{(k,l+1)}] = 0$$

Taking account of (2.7), we obtain an expression for C_{np} from (3.1)

$$C_{np} = -2\pi\,[\rho h a_1 a_2(\omega_{np}{}^2 - \omega^2)]^{-1}(b_{2p}B_{2n} + b_{1n}B_{1p})$$

The first of equations (3.2) yields

$$B_{1p} = -\frac{1}{\alpha_{1p}}\sum_{n=1}^{M}(-1)^n\beta_{np}B_{2n}$$

This last equation results in a system of N equations with N unknowns

$$\alpha_{2n}B_{2n} - \sum_{p=1}^{N}\frac{(-1)^p\beta_{np}}{\alpha_{1p}}\sum_{q=1}^{N}(-1)^q\beta_{qp}B_{2q} = 0 \tag{3.2}$$

Here

$$\alpha_{1q} = \sum_{p=1}^{N}\frac{(-1)^p b_{1p}^2}{\omega_{pq}^2 - \omega^2}, \qquad \alpha_{2p} = \sum_{q=1}^{N}\frac{(-1)^q b_{2q}^2}{\omega_{pq}^2 - \omega^2}$$

$$\beta_{pq} = b_{1p}b_{2q}(\omega_{pq}^2 - \omega^2)^{-1}, \qquad b_{\alpha p} = p a_1 a_2 a_\alpha^{-2}[1 - (-1)^p\lambda_\alpha]$$

From the condition that the determinant of the system (3.2) equals zero we obtain a constraint on the frequency. We obtain other constraints from the conditions $|\lambda_1| = |\lambda_2| = 1$. The frequency spectrum is a band spectrum. Assuming $a_1 = a_2 = a$, $\omega_*{}^2 = \omega^2\pi^{-4}a^4\rho h/D$ for a square cell, we find the boundary of the first frequency band: $2 \leqslant \omega_* \leqslant \omega_{1*}$. The estimate of ω_{1*} depends on the number N. For $N = 5$ we obtain $\omega_{1*} = 4.17$. The exact value of ω^*_1 equals the lowest natural vibrations frequency 3.646 for a cell clamped along the contour.

The construction of the bending moments can be illustrated by an example of cylindrical vibrations. A graph of the bending moment M_{11} is given in Fig. 1 for this cell.

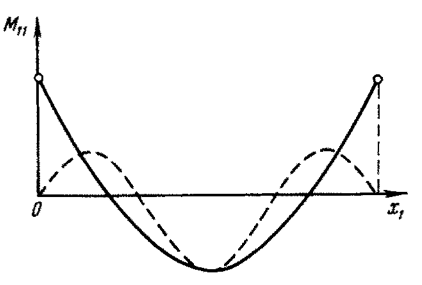

Fig. 1

The continuous line corresponds to the exact solution and the dashes to the approximate solution for $N = 3$. The asterisks yield the values of the edge moments found by means of (2.3). A comparison results in the deduction that far from the perturbation line the state of stress can be found approximately by differentiating the solution. The state of stress can be found by means of (2.3) on the perturbation lines.

4. Let us study the behavior of the factors λ_1, λ_2 for the simplest boundary conditions in the case of a finite doubly-periodic system. The equation for λ_1 will be

$$\lambda_1^2 - 2\lambda_1\gamma_1 + 1 = 0, \quad \gamma_1 = \cos k_1a_1 - \beta_1 k_1^{-1}\sin k_1a_1$$

This equation has two roots (λ_{11} and $\lambda_{12} = \lambda_{11}^{-1}$). For $|\gamma| < 1$ both roots are complex and equal to one in absolute value. Introducing two constants A_{11} and A_{12}, we obtain

$$C_{11}^{(k)} = (\lambda_{11} - \cos k_1a_1)\lambda_{11}^k + (\lambda_{12} - \cos k_1a_1)\lambda_{11}^k A_{12}$$
$$C_{12}^{(k)} = -C_{13}^{(k)} = (\lambda_{11}^k A_{11} + \lambda_{12}^k A_{12}) \sin k_1a_1$$
$$C_{14}^{(k)} = -(\lambda_{11}^{k+1}A_{11} + \lambda_{12}^{k+1}A_{12}) \sin k_1a_1$$

Let two opposite edges be supported. We find that this is equivalent to the conditions $C_{12}^{(1)} = 0$, $C_{14}^{(K)} = 0$. Hence

$$(\lambda_{11}^K - \lambda_{12}^K)\sin^2 k_1a_1 = 0, \quad (\lambda_{11}A_{11} + \lambda_{12}A_{12})\sin k_1a_1 = 0$$

Noting that $\lambda_{11} = \pm 1$ for $\sin k_1a_1 = 0$, we obtain that λ_{11} should satisfy the equation

$$\lambda_{11}^{2K} = 1 \tag{4.1}$$

Let us find the natural vibration frequencies and modes of the part of the structure considered above. Let us consider that the rectangular plate occupies KL cells, such that k varies between one and K, and l between one and L. Moreover, let us assume that each two opposite edges of the plate are either supported or fixed. Let us apply the Bolotin asymptotic method [3] for the solution. Let us use the following expression for the deflection within each cell:

$$w^{(kl)} = [C_{11}^{(k)} \sin k_1x_1' + C_{12}^{(k)} \cos k_1x_1' + C_{13}^{(k)} e^{-\beta_1x_1'} + C_{14}^{(k)} e^{\beta_1(x_1'-a_1)}] \times$$
$$[C_{21}^{(l)} \sin k_2x_2' + C_{22}^{(l)} \cos k_2x_2' + C_{23}^{(l)} e^{-\beta_2x_1'} + C_{24}^{(l)} e^{\beta_2(x_2'-a_2)}]$$
$$\beta_1^2 = k_1^2 + 2k_2^2, \quad \beta_2^2 = 2k_1^2 + k_2^2, \quad \rho h\omega^2 = D(k_1^2 + k_2^2)^2$$

The condition that the deflection on the cell contour equals zero and the matching conditions for the solutions on the cell boundary, yield

$$C_{13}^{(k)} = -C_{12}^{(k)}, \quad C_{14}^{(k)} = -C_{11}^{(k)} \sin k_1a_1 - C_{12}^{(k)} \sin k_1a_1$$
$$C_{12}^{(k+1)} = -C_{14}^{(k)}, \quad C_{11}^{(k+1)} = \cos k_1a_1\, C_{11}^{(k)} - \sin k_1a_1\, C_{12}^{(k)} - \frac{\beta_1}{k_1} C_{14}^{(k)}$$

All the roots of this equation correspond to the natural vibrations modes, with the exception of the case when $\sin k_1a_1 \neq 0$, $\lambda_1 = \pm 1$. The appropriate vibration modes equal zero identically.

Let two opposite edges be fixed. This is equivalent to the conditions

$$k_1C_{11}^{(1)} + \beta_1C_{12}^{(1)} = 0, \qquad \gamma_1C_{11}^{(K)} - (\sin\ k_1a_1 + \beta_1k_1^{-1}\cos\ k_1a_1)\ C_{12}^{(k)} = 0$$

Hence

$$(\lambda_{11} - \lambda_{12})(\lambda_{11}A_{11} - \lambda_{12}A_{12}) = 0, \quad (\lambda_{11} - \lambda_{12})^2(\lambda_{11}{}^K - \lambda_{12}{}^K) = 0$$

Therefore, (4.1) should be satisfied.

All the roots of (4.1), with the exception of the case when $\sin k_1a_1 = 0$ will correspond to the natural vibration frequencies and modes. In this case $C_{1j}^{(K)} = 0$ $(j = 1, 2, 3, 4)$.

There results from (4.1) that λ_1 equals one in absolute value, i.e. it is possible to put $\lambda_1 = \exp(i\varphi_1)$. We obtain from (4.1)

$$K\varphi_1 = m\pi \qquad (m = 0, 1, \ldots, K)$$

Using the equation for λ_1, we find

$$\cos\varphi_1 = \gamma_1 = \cos k_1a_1 - \beta_1k_1^{-1}\sin k_1a_1$$

Analogously, we obtain an equation for the variable x_2

$$\cos\varphi_2 = \cos k_2a_2 - \beta_2k_2^{-1}\sin k_2a_2$$

The results of calculating the lowest group of dimensionless frequencies $\omega_* = \omega a^2\pi^{-2}(\rho h\ /\ D)^{1/2}$ for the case of a square cell $(a_1 = a_2 = a)$ are given in the table (it has the form of a symmetric matrix). In case all the edges are supported, the last column and the last row in the table should be deleted, while the first column and first row should be deleted in the case of clamped contour. If two edges are supported and the others are fixed, then the first column and last row should be omitted.

$K = L = 8$								
2.00	—	—	—	—	—	—	—	—
2.03	2.06	—	—	—	—	—	—	—
2.11	2.14	2.20	—	—	—	—	—	—
2.24	2.27	2.32	2.43	—	—	—	—	—
2.39	2.41	2.45	2.58	2.66	—	—	—	—
2.57	2.59	2.63	2.75	2.88	3.02	—	—	—
2.75	2.77	2.79	2.90	3.03	3.17	3.34	—	—
2.88	2.90	2.93	3.02	3.14	3.28	3.39	3.48	—
2.93	2.95	2.97	3.07	3.19	3.31	3.42	3.52	3.56

5. Let us study the asymptotic properties of the frequencies for a rectangular plate with a large, but finite, number of spans in both directions. Let us use the results of Sect. 4. First, let us note that points on the plane of the wave numbers k_1, k_2 which correspond to the natural vibration frequencies and modes are grouped in domains governed by the inequalities $|\gamma_1| \leqslant 1$, $|\gamma_2| \leqslant 1$. It is interesting to find the density of the frequency numbers of the k_1, k_2 plane. Let some k_1, k_2 satisfy the equations

$$\gamma_1(k_1, k_2) = \cos\frac{\pi p_1}{K}, \qquad \gamma_2(k_1, k_2) = \cos\frac{\pi p_2}{L}$$

Let us find $\Delta_\alpha k_1$ and $\Delta_\alpha k_2$, which correspond to an increase in p_α by one. Then the desired density will approximately equal $n(k_1, k_2) \approx (\Delta_1 k_1 \Delta_2 k_2 - \Delta_1 k_2 \Delta_2 k)^{-1}$ or

$$n(k_1, k_2) \approx |F| KL [(1-\gamma_1^2)(1-\gamma_2^2)]^{-1/2}, \quad F = f_1 f_2 - 4(\beta_1\beta_2)^{-1} s_1 s_2$$
$$f_\alpha = (a_\alpha - 2\beta_\alpha^{-1} q_\alpha^2) s_\alpha + \beta_\alpha k_\alpha^{-1} a_\alpha c_\alpha$$
$$q_1 = k_2 / k_1, \ q_2 = k_1 / k_2, \ s_\alpha = \sin k_\alpha a_\alpha, \ c_\alpha = \cos k_\alpha a_\alpha \ (\alpha = 1, 2)$$

The density of the frequency distribution on the ω axis can be found by integration

$$n(\omega) = \frac{1}{2} \omega^{-1/2} \left(\rho \frac{h}{D}\right)^{1/4} \int_\Gamma n(k_1, k_2)\, d\Gamma$$
$$\Gamma : k_1^2 + k_2^2 = \omega (\rho h / D)^{1/2}; \quad k_{1,2} \geqslant 0, \ |\gamma_{1,2}| \leqslant 1$$

The results of calculating the dimensionless frequency density

$$n_*(\omega) = 4\pi (D / \rho h)^{1/2} (K L a_1 a_2)^{-1} n(\omega)$$

are given in Fig. 2 for a plate with square $(a_1 = a_2)$ cells as a function of the reduced frequency.

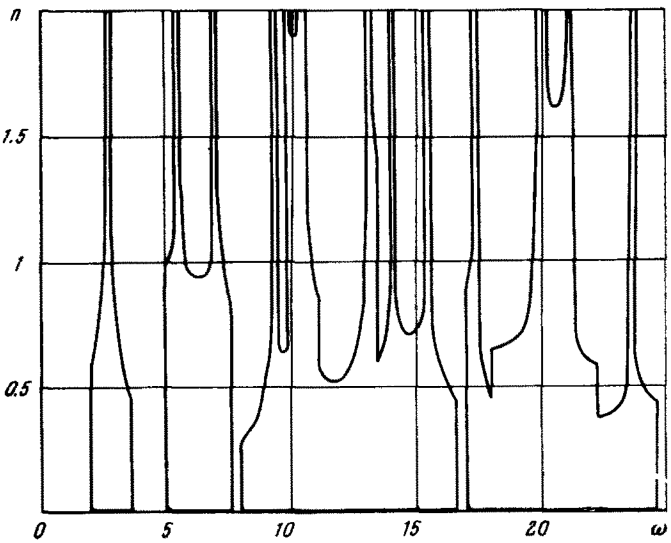

Fig. 2

The frequency spectrum is a banded spectrum. The frequency density receives finite increments at frequencies which are natural for a cell supported along the contour (positive increments) and for a fixed cell(negative). Logarithmic-type singularities exist at frequencies which agree with the natural frequencies for cells supported along two opposite edges and fixed at the other two.

Let us examine frequency bands such that an infinitesimal change in the vibrations mode will correspond to an infinitesimal change in the frequency for each of the bands. In contrast to the case of a beam, the frequency bands for plate can be superposed. For example, five essentially different natural vibrations modes correspond to each frequency on the segment [20, 21, 31].

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THEORY OF AN ELASTIC LINEARLY REINFORCED COMPOSITE

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Elastic fibrous composites with an arbitrary cell microstructure are studied. A procedure is developed for determining the state of stress and the macroscopic properties of such materials. A rigorous foundation is given for the algorithms obtained. Results of computations are presented.

Composites with the simplest cell microstructure have been studied in [1], as well as by the method of [2] in [3]. General methods for investigating elastic inhomogeneous structures are contained in [4, 5].

1. Computational scheme for a composite. Formulation of the problem. Let us consider a three-dimensional isotropic medium reinforced by a doubly-periodic (in the sense of the geometry and elastic characteristics) system of groups of rectilinear fibers with cylindrical cavities (Fig. 1). The geometric and elastic properties of such a medium are described completely by the microstructure of the (fundamental) cell being duplicated periodically. Let us assume that the fibers are set in the medium with some transverse tension, identical at congruent points and constant along the fiber length. The connection between the medium and fiber is such that the force vector varies continuously during passage through the contact boundary, while the displacement vector undergoes a jump due to the transverse tension.

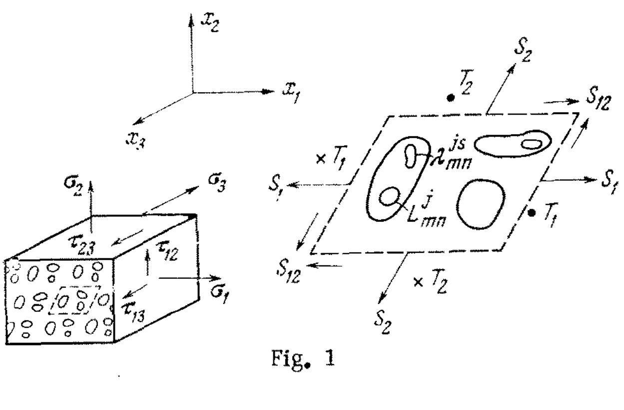

Fig. 1